Critical Embedded Real-Time Systems

Systèmes Temps Réel Embarqués Critiques

STREC - WCET - IPET

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Outline

Sub-Module Outline

1. The `While` Language
2. Basic Data-Flow Analysis
3. **Worst-Case Execution Time Analysis**
   - Approaches (Measurement, Probabilistic, Static Analysis)
   - Flow Facts (Loop Bounds)
   - Pipeline Analysis
   - Implicit Path Enumeration
4. Static Cache Analysis

Worst-Case Execution Time

Worst-Case Execution Time

Real-time systems:

- So far in this course:
  - Scheduling of real-time tasks
  - Each task τ_i has a *Worst-Case Execution Time* C_i (WCET)
  - Each task τ_i has a deadlines (D_i)
  - Can we schedule the whole system?

- Next few sessions:
  - How can we define the WCET ?
  - How can we determine the WCET (C_i)?
  - How long does it take to finish a computation?
    $\Longrightarrow$ We need to analyze (*reason* about) the program!

Worst-Case Execution Time (2)

Some definitions related to timing analysis:

Assume we could observe **all** possible inputs/executions.

Worst-Case Execution Time Bound

Actually, we search for a WCET bound

- Safety:
  A bound is safe when it is *larger* than any observable actual WCET
  $\Longrightarrow$ How can we ensure that the obtained bound is safe?

- Overestimation:
  Imprecision in the analysis lead to overestimation
  $\Longrightarrow$ How can we ensure that the bound is tight?

- From now on: WCET denotes the WCET bound
  WCET . . . WCET bound actual WCET . . . WCET

Factors Impacting the WCET

Factors that may impact the WCET:

- The program source (algorithm)
- The program input (data)
- The compiler (generating machine-level code)
- The hardware platform
  - Processor pipeline
  - Computational units
  - Branch prediction
  - Caches
  - Buffers
  - Main memory
  - Bus arbitration
  - ...
- Other tasks in the system (preemption, competition)

WCET Challenges

What is so difficult with that?

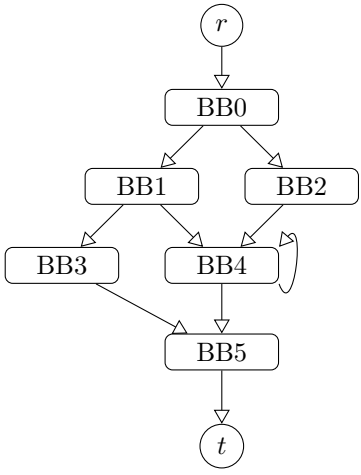
- What is the program doing?
  - Or: which instructions are executed?
  - Depends on algorithms/programing languages/ compilers/...
  - Often also dependent on program inputs
- What are the possible inputs?
  - Usually too many options to explore them all
- How long do the instructions take?
  - Highly dependent on hardware design

WCET Analysis Approaches

Three main approaches:

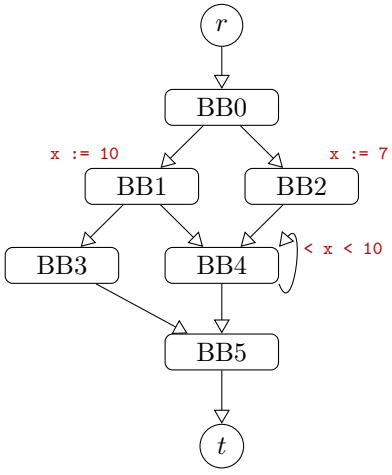
- Measurements: (no guarantee)
  - Simply run the program many times (testing)
  - Covering *all* classes of inputs
  - Covering *all* execution paths
  - Take maximum (multiplied by x)
- Probabilistic Analysis: (requires preconditions)
  - Take measurements (as above)
  - Fit a probabilistic distribution
  - Select WCET subject to a threshold using the distribution
- Static Program Analysis: (generally safe)
  - Analyze code by *abstractions*, e.g., data-flow analysis
  - Extract and annotate information from/to code
  - Safe WCET when abstractions are safe

Example: Static WCET Analysis



Three analysis phases:

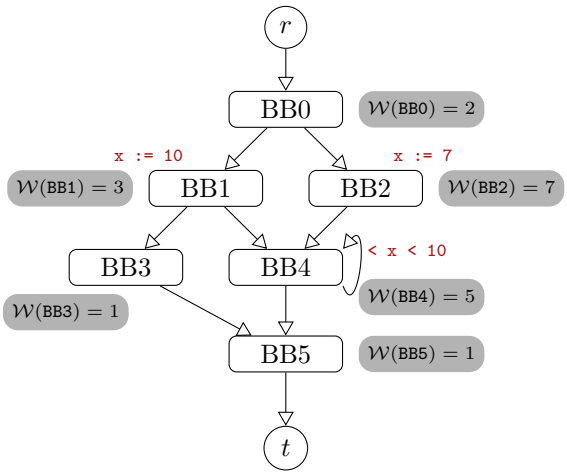
Example: Static WCET Analysis



Three analysis phases:

(1) Loop bounds & flow facts

Example: Static WCET Analysis

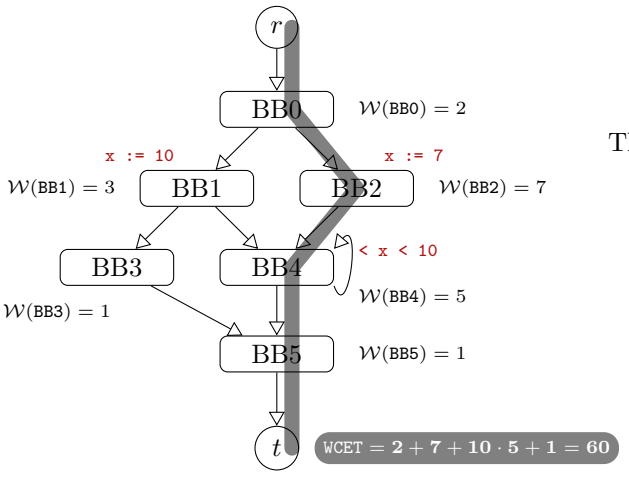


Three analysis phases:

(1) Loop bounds & flow facts

(2) Pipeline & caches

Example: Static WCET Analysis



Three analysis phases:

(1) Loop bounds & flow facts

(2) Pipeline & caches

(3) Longest path search (IPET)

What's next?

- Today:
  - Loop bounds and flow-facts analysis (Step 1)
  - Pipeline analysis (Step 2)
  - Implicit path enumeration (Step 3)
- Next session:
  - Analyzing data/instruction caches (Step 2)
    (Not all of you)

Loop Bounds and Flow Facts

Flow Facts

Information on infeasible program executions:

- Loop bounds:
  The number of iterations of a loop can not exceed a given constant k.
- Recursion bounds:
  May refer to recursion depth (depth of call tree) or number of total recursive calls (number of nodes in the call tree).
- Mutual exclusion:
  Two branch conditions a and b are mutually exclusive, i.e., $a \Rightarrow \neg b$.
- Generic flow facts:
  Relate the execution frequencies of two program points to each other.

Simple Loop Bounds

Trivial analysis for counting loops:

- Easily recognizable patterns (covers most loops)
- Simply take results from range analysis

- Example:

```
for (int i = 0; i < n; i++) {
  ...
}
```

Complex Loop Bounds

Beyond the scope of this course:

- Two major sources of complexity:
  - Complex conditions
  - Nested loops where inner bounds depend on outer loops
- Great challenge for analysis (manual annotations)
  - Former case is equivalent to the halting problem (NP-hard)
  - The later case is well understood
  - Loops in real-time software are typically *well-behaved*

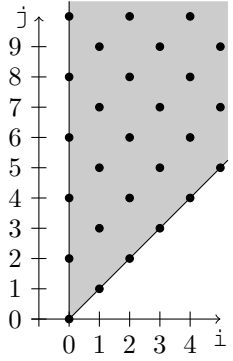
Example: Complex Loops Bounds

Construct linear equations describing *iteration space*

- Equations specify a (parametric) polytope
- Count the number of *integer points* within the polytope

```
for(int i = 0; i < n; i++)
{
  for(int j = i; j < 2*n; j+2)
  {
    ...
  }
}
```

(a) Program code



(b) Corresponding polytope

Pipeline Analysis

Pipeline Analysis

Compute potential states of the processor pipeline:

- Hardware utilization captured using *state machines*
- Abstract interpretation:
  - *Brute force* enumeration of all possible states
  - Sets of pipeline states (Domaine)
  - Compute all potential successor states (Transfer functions)
  - Take union of all states on joins (Meet)
  - Abstractions are difficult due to dynamic pipeline behavior
    $\Longrightarrow$ Interaction with caches, branch prediction, ...
    $\Longrightarrow$ Predictable processors have been proposed[1]

[1] http://patmos.compute.dtu.dk/

Instruction Timing

How do we obtain the instruction timing?

- Consider all states involving a given instruction
  - From the first attempt to fetch the instruction ...
  - To its completion in the pipeline
- Problem:
  - Execution of instructions may overlap
  - Same time instant is *counted several times*
- Solution:
  - Consider basic blocks (sequences of instructions) at once
  - Consider states *in the middle* of control-flow edges
  - Find longest sequence from incoming to outgoing edge
    (longest path search on an acyclic graph)

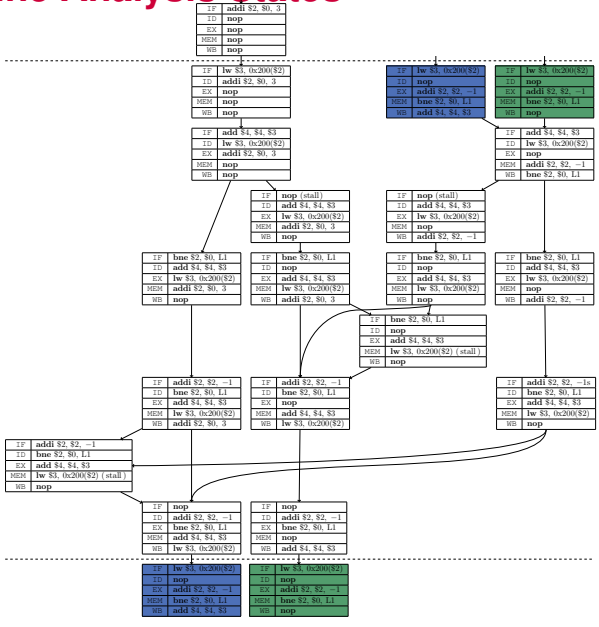
Example: Pipeline Analysis

Assume a pipelined MIPS processor

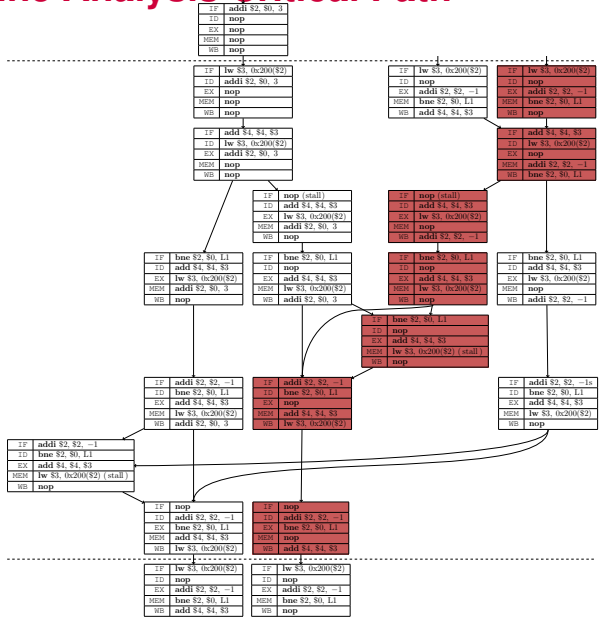
- With 5-stages (IF, ID, EX, MEM, WB)
- Branches execute in EX (2 branch delay slots)
- Instruction and data caches with 16 byte blocks
- IF/MEM are stalled on cache misses for a cycle
- We consider all possible cache states

```
0x14    addi $2, $0, 3
     L1:
0x18    lw $3, 0x200($2)
0x1C    add $4, $4, $3
0x20    bne $2, $0, L1
0x24    addi $2, $2, -1
0x2C    nop
```

Example: Pipeline Analysis States



Example: Pipeline Analysis Critical Path



Limitations

Which cases are covered by the analysis?

- Contiguous execution of the program
  - No interrupts (perturbation of pipeline state)
  - No preemption (requires interrupts)
  - No faults (electric glitches)
  - No operating system calls (often excluded from analysis)
  - No interference in multi-core architectures
- Software correctness
  - Analysis considers all cases right or wrong
  - But does not distinguish between them
  - That is somebody else's problem

Implicit Path Enumeration Technique (aka. IPET)

Bounding the WCET

What have we got so far?

- Analysis of program semantics: (Step 1)
  - Range analysis of program variables
  - Analysis of loop bounds
  - Analysis of generic flow constraints
- Analysis of hardware behavior: (Step 2)
  - Analysis of pipeline states
  - Missing: Caches and branch predictors

Bounding the WCET

What is left to do?

- Actually bounding the WCET

- Problem statement:
  - Find longest execution from program start to its termination
    - Variants: find longest execution of a loop/function/...
  - Equivalent to the **longest paths** in the control-flow graph
    - Nodes of the graph represent basic blocks
    - Edge weights represent basic block execution times (cf. pipeline analysis)

Implict Path Enumeration Technique (IPET)

Build linear equations modeling execution flow:

- Control-flow edges are represented by flow variables
- Flow variables indicate the number of times code executes
- Build a huge linear equation system
  - Solved using standard software (e.g., CPLEX, Gurobi, lpsolve)
  - Maximize execution flows according to edge weights

- Kirchhoff's law:
  The sum of the **flow entering** a control-flow node has to **match** the **flow leaving** the node.

IPET Base Equations

Given a weighted control-flow graph $G = (V, E, \mathcal{W})$ and a mapping of edges to flow variables $\mathcal{F}$:

- Flow for program entry r:

$$\sum_{(r,n)\in E} \mathcal{F}(r,n) = 1$$

- Flow for program exit t:

$$\sum_{(n,t)\in E} \mathcal{F}(n,t) = 1$$

- Flow equations of node $n \in V$:

$$\forall n \in V: \sum_{(k,n)\in E} \mathcal{F}(k,n) = \sum_{(n,m)\in E} \mathcal{F}(n,m)$$

- **Maximizing**:

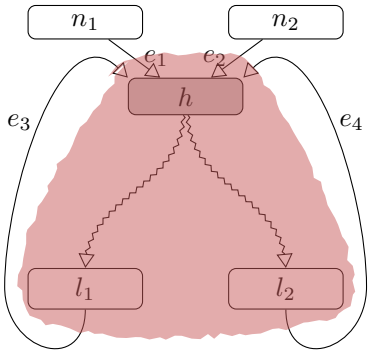
$$max. \sum_{(m,n)\in E} \mathcal{F}(m,n) \cdot \mathcal{W}(m,n)$$

Loop Bounds in IPET

Given a reducible loop L with bound $\hat{b}$ and loop header h:

$$\sum_{(n,h)\in E} \mathcal{F}(n,h) \leq \hat{b} \cdot \sum_{(n,h)\notin L} \mathcal{F}(n,h)$$

Example:

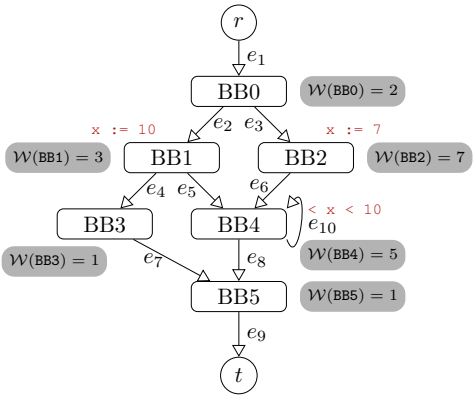


- Loop: $L = \{h, \ldots, l_1, l_2\}$ (red)
- Header: h (darker node)
- Pre-entries: $n_1, n_2 \notin L$

- Equations:

$$e_1 + e_2 + e_3 + e_4 \leq \hat{b} \cdot (e_1 + e_2)$$

Example: IPET

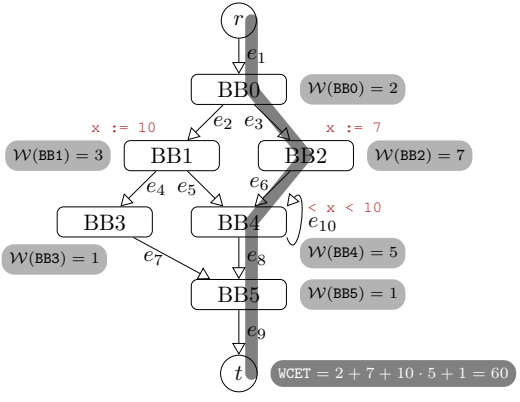


$$
\begin{aligned}
e_1 &= 1 \\
e_1 &= e_2 + e_3 \\
e_2 &= e_4 + e_5 \\
e_3 &= e_6 \\
e_4 &= e_7 \\
e_5 + e_6 + e_{10} &= e_8 + e_{10} \\
e_7 + e_8 &= e_9 \\
e_9 &= 1
\end{aligned}
$$

$$
e_5 + e_6 + e_{10} \leq 10 \cdot (e_5 + e_6)
$$

$$
\textbf{Maximize} : 2e_2 + 2e_3 + 3e_4 + 3e_5 + 7e_6 + e_7 + 5e_8 + e_9 + 5e_{10}
$$

Example: IPET (2)



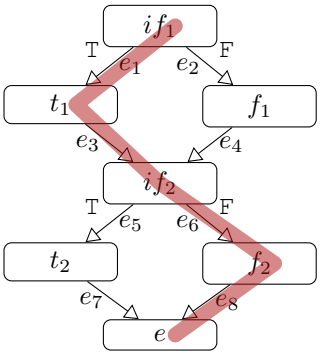
$$1 = 1$$
$$1 = 0 + 1$$
$$0 = 0 + 0$$
$$1 = 1$$
$$0 = 0$$
$$0 + 1 + 9 = 1 + 9$$
$$0 + 1 = 1$$
$$1 = 1$$

$$0 + 1 + 9 \leq 10 \cdot (0 + 1)$$

Maximize : $2 \cdot 0 + 2 \cdot 1 + 3 \cdot 0 + 3 \cdot 0 + 7 \cdot 1 + 0 + 5 \cdot 1 + 1 + 5 \cdot 9$

Group Exercise: Infeasible Paths in IPET

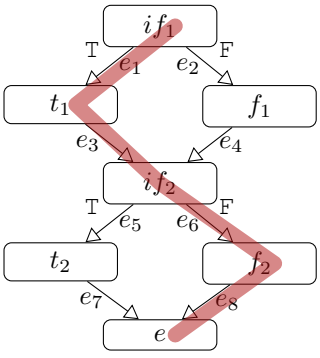
Determine the equations to exclude the highlighted path:



- Assume that the in-flow of if_1 might be larger than 1
- Hint:
  Think about the flows related to node if_2

Group Exercise: Infeasible Paths in IPET

Determine the equations to exclude the highlighted path:



- Assume that the in-flow of if_1 might be larger than 1
- Hint:
  Think about the flows related to node if_2

- Solution:

$$e_6 \leq e_4$$

Summary

- Worst-case execution time
  - Bounds vs. actual WCET
  - Overestimation
- Obtaining WCET estimations
  - Static program analysis (guaranteed safe)
  - Measurements (safety not guaranteed)
  - Probabilistic analysis (some prerequisites)
- Static WCET analysis
  - Based on data-flow analysis/abstract interpretation
  - Value range analysis (software behavior)
  - Pipeline analysis (hardware behavior)
  - Implicit path enumeration (compute WCET)