

#### **Critical Embedded Real-Time Systems**

Systèmes Temps Réel Embarqués Critiques

STREC - WCET - Cache Analysis

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#### Outline

### **Sub-Module Outline**

- 1. The While Language
- 2. Basic Data-Flow Analysis
- 3. Worst-Case Execution Time Analysis

#### 4. Static Cache Analysis

- Recap Caches (3Cs, Associativity, Replacement Policy)
- Hit & Miss Classification (May vs. Must Analysis)
- Classification & IPET
- Persistence



## **Cache Organization**

# **Cache Principles**

What is a cache?

- A relatively small and fast memory
- · Connected to a larger and slower cache/memory
- Stores data (or instructions) currently used
  - Implemented as a kind of dictionary
  - Cache hit:

Data requested by the processor is in the cache  $\implies$  Immediate response

#### • Cache miss:

Data requested by the processor is not in the cache

- $\implies$  Data is fetched from larger cache/memory
- $\implies$  Delayed response



#### **Cache Misses**

Sources of misses can be grouped in three categories:

#### • Compulsory misses:

Occur when new data is accessed that was never referenced before

#### • Capacity misses:

Occur due to the limited size of the cache, regardless of the cache's internal design

(i.e., the amount of data accessed is larger than the cache)

#### • Conflict misses:

Are due to the internal organization of the cache

(i.e., could theoretically be avoided by an ideal cache design)



# **Cache Design**

A cache can be seen as a kind of dictionary with *k* entries:

- · Each entry is associated with the following information
  - Valid flag: Flag indicating whether the data is valid
  - Tag: The address of the data held by the entry
  - <u>Data:</u> The data held by the entry
- Entries are stored in a memory
- Cache accesses to address a:
  - 1. Check whether an entry's tag matches a
  - 2. Check whether that entry is valid
  - 3. Yes?  $\Longrightarrow$  hit
  - 4. No?  $\implies$  miss



#### **Set-Associative Cache**

Organize cache in lines to reduce conflicts:

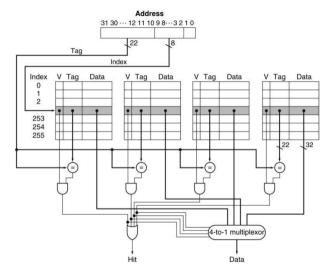
- The cache contains *k* entries
- Each line contains a set of s entries
- Each entry holds a block of b bytes
- Address a maps to a line:

$$(a \div b) \mod (k \div s)$$

- Cache look-up:
  - Read line  $(a \div b) \mod (k \div s)$  from cache
  - Compare the tags of the line's s entries with a
  - Select the matching entry (if one exists)
  - Check the entry's valid flag



# **Set-Associative Cache (2)**



Example: A 4-way set-associative cache.



# **Replacement Policy**

Which entry should be used on a cache miss?<sup>1</sup>

- Several policies are possible
- First-In, First-Out:
  - Simple to implement
  - Replace the entry that was loaded first
  - aka: Round-Robin
- Least-Recently Used (LRU):
  - Widely used strategy in practice (rather expensive through)
  - · Replace block that was not used the longest
  - Preserve blocks that have recently been used (cf. temporal locality)

• . . .



<sup>&</sup>lt;sup>1</sup>Especially once all valid flags are set.

# Least-Recently Used

Implemented as an age counter for each entry:

- Counters are updated on each access to an address a
- Counters are in the range  $[0, 1, \ldots, s-1]$
- <u>Hit:</u>
  - 1. If the age of a's entry is 0: done
  - 2. Otherwise: set the age of that entry to 0
  - 3. Increment the age of the line's other entries by 1
- Miss:
  - 1. Fetch data from backing store
  - 2. Select entry with age s 1
  - 3. Set the counter of that entry to 0
  - 4. Set the valid flag, the tag, and the data accordingly
  - 5. Increment the age of the line's other entries by 1



#### **Example: LRU Replacement**

Cache state when performing memory accesses:

- Assume the following set-associative cache:
  - <u>Block size</u>:  $b = 2^0 = 1$  byte
  - Entries:  $k = 2^3 = 8$
  - Associativity:  $s = 2^1 = 2$
  - Address Width: 5 bits
- The cache is initially empty (i.e., all valid flags are 0)
- Accessed addresses:

22, 26, 22, 26, 16, 3, 16, 18, 26



# Example: LRU Replacement (1)

			Set 0				Set 1	
Index	Valid	Age	Tag	Data	Valid	Age	Tag	Data
0	0	-	_	-	0	-	_	_
1	0	-	_	_	0	—	_	_
2	0	_	_	_	0	—	_	_
3	0	—	—	_	0	—	-	-

Initially: Cache is entirely empty.



# Example: LRU Replacement (2)

			Set 0				Set 1	
Index	Valid	Age	Tag	Data	Valid	Age	Tag	Data
0	0	_	_	_	0	-	_	_
1	0	_	_	_	0	_	_	_
2	1	0	101 <sub>2</sub>	<i>M</i> (10110 <sub>2</sub> )	0	—	_	_
3	0	—	-	-	0	—	-	_

Miss: Compulsory miss for address 22.



# Example: LRU Replacement (3)

			Set 0				Set 1	
Index	Valid	Age	Tag	Data	Valid	Age	Tag	Data
0	0	_	_	_	0	_	-	_
1	0	_	_	_	0	_	_	_
2	1	1	101 <sub>2</sub>	<i>M</i> (10110 <sub>2</sub> )	1	0	110 <sub>2</sub>	<i>M</i> (11010 <sub>2</sub> )
3	0	_	-	_	0	_	_	-

Miss: Compulsory miss for address 26. (same line, but no conflict)



# Example: LRU Replacement (4)

			Set 0				Set 1	
Index	Valid	Age	Tag	Data	Valid	Age	Tag	Data
0	1	0	100 <sub>2</sub>	$M(10000_2)$	0	_	—	_
1	0	_	_	_	0	_	_	_
2	1	1	101 <sub>2</sub>	<i>M</i> (10110 <sub>2</sub> )	1	0	110 <sub>2</sub>	$M(11010_2)$
3	0	—	-	_	0	_	-	_

<u>Hits:</u> Cache hits for addresses 22 and 26. (intermittently switch age) <u>Miss:</u> Compulsory miss for address 16.



# **Example: LRU Replacement (5)**

			Set 0				Set 1	
Index	Valid	Age	Tag	Data	Valid	Age	Tag	Data
0	1	0	100 <sub>2</sub>	$M(10000_2)$	0	_	—	_
1	0	_	_	_	0	_	_	_
2	1	1	101 <sub>2</sub>	<i>M</i> (10110 <sub>2</sub> )	1	0	110 <sub>2</sub>	<i>M</i> (11010 <sub>2</sub> )
3	1	0	0002	<i>M</i> (00011 <sub>2</sub> )	0	—	-	-

Miss: Compulsory miss for address 3.



# **Example: LRU Replacement (6)**

			Set 0				Set 1	
Index	Valid	Age	Tag	Data	Valid	Age	Tag	Data
0	1	0	100 <sub>2</sub>	$M(10000_2)$	0	_	—	_
1	0	_	_	_	0	_	_	_
2	1	0	100 <sub>2</sub>	<i>M</i> (10010 <sub>2</sub> )	1	1	110 <sub>2</sub>	$M(11010_2)$
3	1	0	0002	<i>M</i> (00011 <sub>2</sub> )	0	—	-	_

<u>Hit:</u> Cache hit for address 16. <u>Miss:</u> Compulsory miss for address 18. (conflict with addresses 22)



# Example: LRU Replacement (7)

			Set 0				Set 1	
Index	Valid	Age	Tag	Data	Valid	Age	Tag	Data
0	1	0	100 <sub>2</sub>	<i>M</i> (10000 <sub>2</sub> )	0	_	—	_
1	0	_	_	_	0	_	_	_
2	1	1	100 <sub>2</sub>	<i>M</i> (10010 <sub>2</sub> )	1	0	110 <sub>2</sub>	<i>M</i> (11010 <sub>2</sub> )
3	1	0	0002	<i>M</i> (00011 <sub>2</sub> )	0	-	-	

Hit: Cache hit for address 26.



# Write Policy (Hit)

Determines how memory stores are handled:

• Two basic options for a write hit

#### • Write-through:

- Write data into the cache and to backing store
- Long delay (waiting for slow higher-level caches)

#### • Write-back:

- Write data only to the cache
- Data is incoherent between cache and backing store
- Backing store updated once data is evicted from cache
- Implementation:

Add an additional dirty bit to each cache entry

• What happens on a write miss?



# Write Policy (Miss)

Should data be loaded to the cache on a write miss?

- Write-allocate:
  - · First load cache block from backing store
  - Then use same strategy as for write hits
- Write-no-allocate:
  - · Does not load from backing store
  - · Write immediately to backing store
- · Both can be combined with write-trough/-back, but usually
  - Write-through is combined with write-no-allocate
  - Write-back is combined with write-allocate



#### **This Course**

From now on we will assume:

- Separate data and instruction caches
- LRU replacement policy
- Write-through with write-no-allocate



# **Cache Analysis**

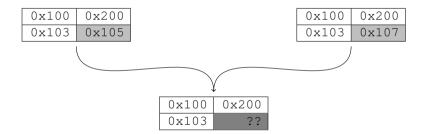
#### **Cache Analysis**

Compute the time required for cache misses:

- Analyze cache states before each memory access
  - Is the accessed data in the cache?
  - How often do cache hits occur?
  - · How often do the expensive cache misses occur?
- Problems:
  - Access addresses need to be known precisely
  - Behavior of accesses in loops changes over time



Combining two cache states (addresses)\*



\*Cache configuration 2-way set-associative, 1 word blocks, 2 cache lines, LRU replacement



Initial cache state (addresses)\*

0x100	0x200
0x103	??

\* Cache configuration 26/1 2-way set-associative, 1 word blocks, 2 cache lines, LRU replacement



Initial cache state (addresses)\*

0x100	0x200
0x103	??

lw	[0x100]	
тw	[UXIUU]	

0x100	0x200
0x103	??

Classified as hit

\* Cache configuration 26/1 2-way set-associative, 1 word blocks, 2 cache lines, LRU replacement



Initial cache state (addresses)\*

0x100	0x200
0x103	??

<b>lw</b> [0x100]	0x100 0x103	0x200 ??	Classified as hit
<b>lw</b> [0x300]	0x300 0x103	0x100 ??	Classified as miss

\* Cache configuration 26/1 2-way set-associative, 1 word blocks, 2 cache lines, LRU replacement



Initial cache state (addresses)\*

0x100	0x200
0x103	??

<b>lw</b> [0x100]	0x100 0x103	0x200 ??	Classified as hit
<b>lw</b> [0x300]	0x300 0x103	0x100 ??	Classified as miss
<b>lw</b> [0x105]	0x100 0x105	0x200 ??	Classification unclear

\*Cache configuration 26/1 2-way set-associative, 1 word blocks, 2 cache lines, LRU replacement



Initial cache state (addresses)\*

0x100	0x200
0x103	??

<b>lw</b> [0x100]	0x100	0x200	Classified as hit
<b>lw</b> [0x100]	0x103	??	Classilled as fill
<b>lw</b> [0x300]	0x300	0x100	Classified as miss
	0x103	??	
<b>lw</b> [0x105]	0x100	0x200	Classification unclear
	0x105	??	Olassification unclear
<b>lw</b> [??]	??	??	Classification unclear
	??	??	Classification unclear



\*Cache configuration

2-way set-associative, 1 word blocks, 2 cache lines, LRU replacement 26/1

#### **Cache Hit & Miss Classification**

#### **Basic Idea**

For each memory/cache access:<sup>2</sup>

- 1. Determine the set of memory blocks potentially accessed
  - For instance: range analysis (last lecture)
- 2. Determine the age of each memory block
  - Topic of today's lecture
- 3. Use the minimum/maximum age to classify hits/misses
  - Topic of today's lecture



<sup>&</sup>lt;sup>2</sup>Recall: assume a set-associative cache with LRU

### **Memory Blocks**

Abstraction used during the analysis to track the cache state:

- Address range in memory corresponding to a cache block
- Aligned with the cache block size
- Matches the size of a cache block (b from above)
- <u>Notations:</u>
  - *mb*<sub>*l*</sub>(*i*) denotes the set of memory blocks of cache line *l*, potentially accessed by instruction *i*
  - This set might be empty

(e.g., instructions not accessing memory, such as  ${\tt addi}$  on MIPS)



Associate each memory block with an age counter:

- Counter range: [0, 1, ..., *s*]
- Difference with *real* cache:
  - Track age of all memory blocks not just those in the cache
  - Memory blocks that are not in the cache have age *s* (compare with counter range of actual cache)
- Notations:
  - *age*(*u*) denotes the age of memory block *u*.



(*s* entries per set)

#### **Access Classification**

Classify each memory/cache access into a category:

• Always Hit (AH):

The age of all potentially accessed memory blocks <u>must</u> be smaller than *s*.

• Always Miss (AM):

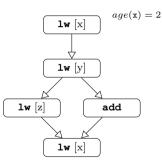
The age of any potentially accessed memory blocks may never be smaller than *s*.

• Not classified (NC): None of the above applies.



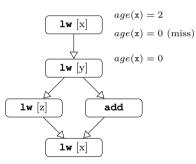
#### Example: Age-Based Cache Analysis

Computing the age of cache block x:



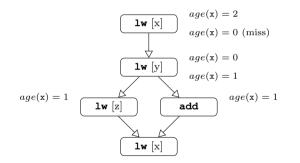


Computing the age of cache block  $\times$ :



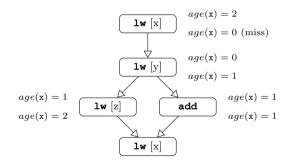


Computing the age of cache block x:



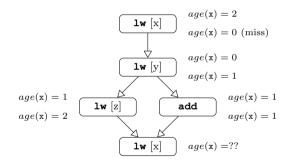


Computing the age of cache block x:





Computing the age of cache block x:

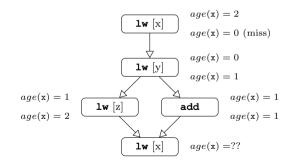




#### Groupe Exercise Age-Based Cache Analysis

What is the classification of the last access to x?

- What can be said about the age of the memory block?
- Hint: recall the words may and must in the category definitions





The above classification gives rise to two analysis problems

#### • Must analysis:

(pessimist)

Compute maximum age of memory blocks, i.e., ages appearing in any real execution **must** be equal or smaller to the computed age.

#### • May analysis:

(optimist)

Compute minimum age of memory blocks, i.e., there **may** exist a real execution with an age as low as the computed age.



## **Must Analysis**

Data-flow analysis computes maximum age of memory blocks:

- Domain:
  - $CS = MB_I \times \{0, 1, \dots, s\}$
  - MB<sub>I</sub> denotes the set of memory blocks of a cache line I
  - s denotes the number of cache sets
- Notations:
  - *age*(*c*, *u*) gives the age of memory block *u* for cache state *c*
  - Only memory blocks *in* the cache will be shown (i.e., only those with an age smaller than *s*)



## Must Analysis: Join Operator (⊔<sub>MUST</sub>)

Select the maximum age for each memory block from cache states  $c_1, c_2 \in CS$ :

$$c_1 \sqcup_{MUST} c_2 = \{(u, a) | \exists (u, a_1) \in c_1, (u, a_2) \in c_2 \colon a = \max(a_1, a_2)\}$$



# Must Analysis: Transfer Function (1)

Lets consider a single memory block for now:

- Assume a state  $c \in CS$  and a memory block  $u \in MB_I$
- The cache state after a memory load is then given by:

 $update_{MUST}(c, u) = \{(v, a) | v \in MB_l : a = must\_age(c, u, v)\}$ 

$$\textit{must\_age}(c, u, v) = \begin{cases} 0 & , \text{ if } u = v \\ \textit{age}(c, v) & , \text{ if } \textit{age}(c, v) \geq \textit{age}(c, u) \\ \textit{age}(c, v) + 1 & , \text{ if } \textit{age}(c, v) < \textit{age}(c, u) \end{cases}$$

• Memory stores do not impact the cache state (write-through, write-no-allocate)

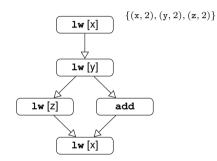


## Must Analysis: Transfer Function (2)

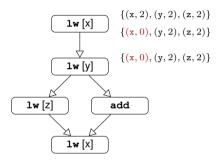
The transfer function for cache state *c* and instruction *i* then is:

$$t_{MUST}(c,i) = \begin{cases} c & , \text{ if } mb_l(i) = \emptyset \\ update_{MUST}(c,u) & , \text{ if } mb_l(i) = \{u\} \\ error & , \text{ otherwise (not yet handled)} \end{cases}$$

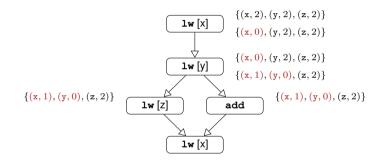




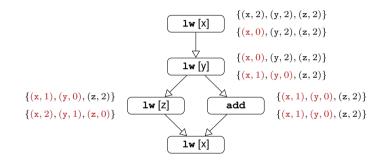




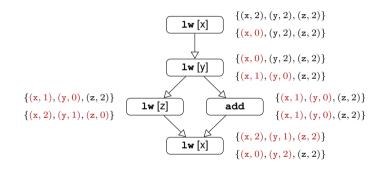










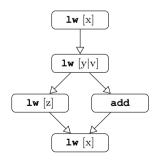




### Group Exercise: Must Cache Analysis

What if the accessed memory blocks are not precisely know?

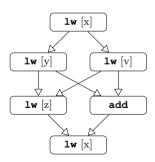
- Assume that the second load might either access  ${\rm y}$  or  ${\rm v}$  (but of course never both)
- Is information regarding other memory blocks lost?





## **Representing Uncertain Accesses**

- Can be seen as a form of *control-flow* decision
- Simply handle both cases separately
- Then apply the join operator
- Example:



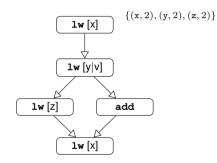


## Must Analysis: Transfer Function (3)

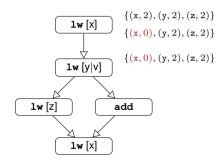
The transfer function for cache state *c* and instruction *i* then is:

$$t_{MUST}(c, i) = \begin{cases} c , & \text{, if } mb_l(i) = \emptyset \\ \bigsqcup_{u \in mb_l(i)} update_{MUST}(c, u) , & \text{, otherwise} \end{cases}$$

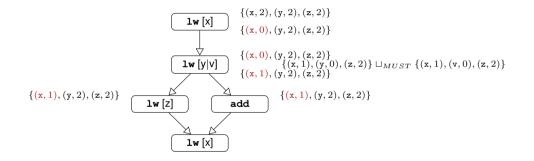




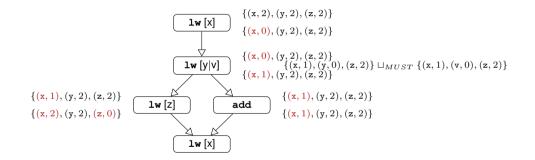




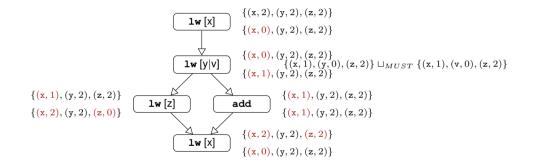














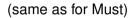
#### May Analysis

Similar data-flow analysis as the Must analysis before:

- Domain:
  - $CS = MB_l \times \{0, 1, \ldots, s\}$
  - MB<sub>I</sub> denotes the set of memory blocks of cache line I
  - s denotes the number of cache sets
- Join Operator:

For any  $c_1, c_2 \in CS$  the join operator is given by:

 $c_1 \sqcup_{MAY} c_2 = \{(u, a) | \exists (u, a_1) \in c_1, (u, a_2) \in c_2 \colon a = \min(a_1, a_2) \}$ 





## May Analysis: Transfer Function

Again, first consider a single memory block:

- Assume a state  $c \in CS$  and a memory block  $u \in MB_l$
- The cache state after a memory load is then given by:

$$update_{MAY}(c, u) = \{(v, a) | v \in MB_l : a = may\_age(c, u, v)\}$$

$$may\_age(c, u, v) = \begin{cases} 0 & , \text{ if } u = v \\ age(c, v) & , \text{ if } age(c, v) > age(c, u) \\ age(c, v) + 1 & , \text{ if } age(c, v) \le age(c, u) \land \\ & & age(c, v) < s \\ s & , \text{ otherwise} \end{cases}$$

- Memory stores do not impact the cache state (as before, write-through, write-no-allocate)
- The actual transfer function is similar to the Must analysis



### Group Exercise: May versus Must Analysis

The age functions of the May and Must analyses are similar:

- Try to explain the differences
- <u>Hint:</u> Recall that the Must analysis provides a maximum and the May analysis a minimum age!

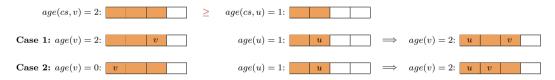
$$must\_age(c, u, v) = \begin{cases} 0 & , \text{ if } u = v \\ age(c, v) & , \text{ if } age(c, v) \ge age(c, u) \\ age(c, v) + 1 & , \text{ if } age(c, v) < age(c, u) \end{cases}$$
$$may\_age(c, u, v) = \begin{cases} 0 & , \text{ if } u = v \\ age(c, v) & , \text{ if } age(c, v) > age(c, u) \\ age(c, v) + 1 & , \text{ if } age(c, v) \ge age(c, u) \\ age(c, v) + 1 & , \text{ if } age(c, v) \le age(c, u) \land age(c, v) < s \\ s & , \text{ otherwise} \end{cases}$$



### May versus Must Analysis

Must analysis:

- age(c, v) represents the **maximum age**, i.e., the actual age might be **smaller**.
- Due to  $age(c, v) \ge age(c, u)$ , the access to u cannot increase the age of v.



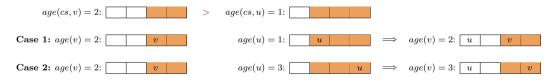
• Similar argument when age(c, v) < age(c, u).



# May versus Must Analysis (2)

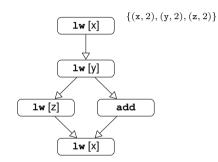
May analysis:

- age(c, v) represents the **minimum age**, i.e., the actual age might be **larger**.
- Due to age(c, v) > age(c, u) the access to u thus cannot increase the age of v

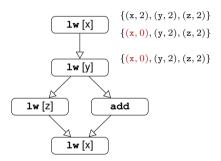


 Similar argument when age(c, v) ≤ age(c, u) (attention age(c, v) might become too large here)

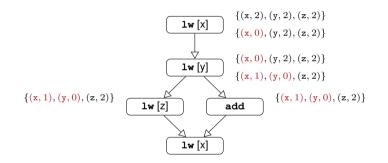




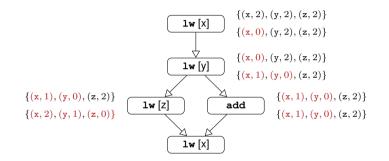




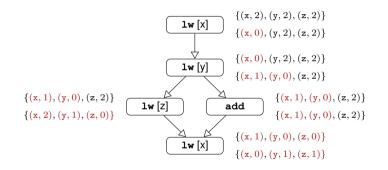














### **Final Classification**

Classification derived from May/Must analyses:

- Always Hit (AH): Must analysis age of memory blocks in mb<sub>l</sub>(i) has to be lower than s.
- Always Miss (AM):

May analysis age of memory blocks in  $mb_l(i)$  has to be equal to s.

• Not classified (NC): None of the above applies.



Access classifications are easy to integrate into IPET

• Always Hit (AH):

Usually does not require additional costs.

• Always Miss (AM):

Add miss costs to the weight of the instruction's basic block

• Not classified (NC):

Often considered as expensive as a miss.<sup>3</sup>

<sup>&</sup>lt;sup>3</sup>This is only safe on architectures without timing anomalies (out of scope of this course – see SE201 at Télécom ParisTech to get an idea)



#### **Persistence**

### **Accesses in Loops**

Behavior of realistic programs:

- · Often repeatedly access the same data in loops
- Observation:
  - First iteration: cache miss
  - · Other iterations: often hits in cache
  - The executed code itself exhibits typically this behavior (instruction cache)

#### Problem:

This is cannot be handled by simple hit/miss classification.

(compulsory miss)



#### Persistence

Introduce the notion of *persistence*:

- Data that remains in the cache once loaded
- Typically with regard to a scope (e.g., a loop, a function, ...)
- <u>New classification:</u>
  - First Miss
  - · Accounting for one miss each time the scope is entered



### Persistence Analysis (Idea)

Determine persistent memory accesses within a scope:

- Various possible approaches
  - · Combine loop peeling with Must analysis
  - Bound set of conflicting accesses
  - ...
- Typically focus on loops
  - Here in particular loop nests



## Summary

- Caches hide long memory access latencies
  - Considerably improve average-case execution time
  - Need to be considered during WCET analysis
- Cache design
  - · Organized in sets of fixed-sized cache blocks
  - Replacement policy (Least Recently Used)
  - Write strategy (Write-through, no-allocation)
- Cache analysis
  - Classify memory accesses (Always Hit/Miss, Not Classified)
  - Must analysis: cache blocks that *must* be in the cache
  - May analysis: cache blocks that may be in the cache

